Solving for a Variable using Factoring

Steps	**Example:**
1. Move all the terms to one side and set them equal to 0 first.	Starting equation: $6x^3 - 12x = -21x^2$ Add $21x^2$ to both sides. $6x^3 + 21x^2 - 12x = 0$
2. Factor out a common factor if there is one.	There is a common factor and it is $3x$. $3x(2x^2 + 7x - 4) = 0$
3. Factor using patterns or rules below.	In this case, the factor $(2x^2 + 7x - 4)$ can still be factored into $(2x-1)(x+4)$ using the "ac method" (see Factoring Techniques below). The equation becomes $3x(2x-1)(x+4) = 0$.
4. Set each factor equal to 0 and solve for each solution of the variable.	$3x = 0$ therefore $x = 0$ $2x - 1 = 0$ therefore $x = 1/2$ $x + 4 = 0$ therefore $x = -4$

Common Factoring Patterns

Sum squared	$(a+b)^2 = a^2 + 2ab + b^2$
Difference squared	$(a-b)^2 = a^2 - 2ab + b^2$
Difference of squares	$a^2 - b^2 = (a-b)(a+b)$
Sum of Cubes	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
	The signs (+ or -) are the *same*, *opposite*, *plus (SOP)* for the factored sum of cubes and difference of cubes.
Difference of Cubes	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Factoring Techniques for $ax^2 + bx + c$

Situation	**Example**
If a = 1 (the first coefficient is 1), find the factors of c that add up to b.	Starting expression: $x^2 - 3x - 10$ c = -10 b = -3 Factors of -10 are: 1 and -10 -1 and 10 2 and -5 2 and -5 Since only 2 and -5 add up to -3, use those value. $x^2 - 3x - 10 = (x+2)(x-5)$

If a ≠ 1, use the "ac method." Multiply a by c, and find the factors of ac that add up to b. Split the middle term into a sum using the two factors as coefficients. Take out the greatest common factors of the two left terms, and the two right terms. The expressions left in the parenthesis should be the same. Place the two terms not in the parenthesis together in a parenthesis as one factor, multiplied by the other term in parenthesis.	Starting expression: $2x^2 - 7x - 15$ 2 x -15 = -30 Factors of -30 that add up to -7 are 3 and -10. Turn -7x into 3x - 10x $2x^2 - 7x - 15$ becomes $2x^2 + 3x - 10x - 15$ $x(2x+3) - 5(2x+3)$ $(x-5)(2x+3)$
If the leading coefficient is negative, factor out -1 first to make it easier to factor.	$-x^2 + 2x + 24$ Factor out -1 $-(x^2 - 2x - 24)$ Continue to factor if possible $-(x-6)(x+4)$
If the expression or equation has a quadratic form but the degree is not 2 (in other words, the leading term is not some multiple of x^2), use u-substitution. Set u equal to the middle term's variable portion. Rewrite the equation in terms of u. Factor the new equation. Solve for u. Substitute what u was originally set equal to in order to solve for the solutions of the original equation.	$x^4 - 13x^2 + 36 = 0$ $u = x^2$ $u^2 - 13u + 36 = 0$ $(u-4)(u-9) = 0$ $u = 4$ $u = 9$ Since u was set equal to x^2, plug x^2 in for u to solve for x. $x^2 = 4$ therefore $x = \pm 2$ $x^2 = 9$ therefore $x = \pm 3$